

# Technical Notes

## Empirical Equation for Self-Starting Limit of Supersonic Inlets

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### Nomenclature

$A$  = area,  $m^2$   
 $M$  = Mach number  
 $\gamma$  = specific heat ratio

### Subscripts

$c$  = cowl-lip section  
 $t$  = inlet throat section

### I. Introduction

FOR hypersonic inlets that operate over a large Mach number range and use a combination of external and internal compression, it must be ensured that inlets can self-start at takeover Mach number. An unstarted inlet captures less airflow with lower efficiency and higher aerodynamic and thermal loads compared with a started inlet. The starting problem was recently studied by many researchers [1–3].

The term started is used to denote operation under conditions where flow phenomena in the internal portion of the inlet do not alter the capture characteristics of the inlet [4]. An inlet can be unstarted by three reasons [4,5]:

- 1) The internal contraction is so large that the inlet is over-contracted to the point where the flow chokes at the inlet throat.
- 2) The back pressure is raised beyond the level that can be sustained by the inlet.
- 3) The formation of a large separated flow makes the flow choked. At high Mach numbers, the unstarted flowfield is usually characterized by a large separated region.

Self-starting indicates that the inlet will start without a change in inlet geometry. It is a significant feature in a fixed-geometry inlet design because it indicates that variable geometry is not necessary to restart the inlet. Preliminary estimates of the internal contraction ratio that will self-start can be obtained from the Kantrowitz limit [7]. This limit is determined by assuming a normal shock at the beginning of the internal contraction and calculating the one-dimensional isentropic internal area ratio that will produce sonic flow at the throat.

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For a perfect gas the internal contraction ratio of Kantrowitz limit can be calculated as follows:

$$\left(\frac{A_c}{A_t}\right)_{\text{Kantrowitz}} = \sqrt{\frac{\gamma + 1}{2 + (\gamma - 1)M_c^2}} M_c^{(\gamma+1)/(\gamma-1)} \cdot \left(\frac{2\gamma}{\gamma+1} M_c^2 - \frac{\gamma-1}{\gamma+1}\right)^{1/(1-\gamma)} \quad (1)$$

where  $A_c$  denotes the area of cowl-lip section,  $A_t$  denotes the area of throat section, and  $M_c$  denotes the average Mach number of cowl-lip section.

According to some experimental data, van Wie [4] pointed out that the Kantrowitz starting limit becomes conservative at high speeds as a result of the assumption of a single normal shock, because the shock system of an unstarted hypersonic inlet has a higher pressure recovery compared with a normal shock. This conclusion is proved by more self-starting testing in recent several years. An empirical equation for self-starting limit of supersonic inlets is attempted to conclude based on these testing data.

### II. Result

The self-starting characteristics of many inlets have been investigated by wind-tunnel testing. The self-starting ability of an inlet is usually proved by the restart of the inlet after a mechanically imposed unstart. These published experimental results are presented in Fig. 1. It must be revealed that some inlet obtained a much larger self-starting internal contraction ratio than Kantrowitz limit by many ways including the use of bleed, bypass ducts, fluid injection, and so on [8]; those experimental data are not plotted in Fig. 1. Furthermore, two self-unstarted experimental values are also supplied in this figure.

The testing presented in Fig. 1 includes three kinds of inlets: two-dimensional (2-D) planar inlet [5,9], sidewall compression inlet [10], and inward turning inlet [11–14]. They imposed unstart mechanically by three ways, corresponding to the three previously mentioned unstarted reasons, respectively: 1) increasing the internal contraction ratio by variable geometry [10,12], 2) raising the back pressure [9,11,13], and 3) increasing cowl-lip shock strength to produce large separation flow [5].

In [5,9,10,12], the maximum self-starting internal contraction ratio for a specific inlet at a certain freestream Mach number was obtained by translating or rotating cowl, respectively. Among of them, [5,12] have close average Mach number of cowl lip, but their maximum self-starting internal contraction ratios have a large difference. This result is caused probably by their different inlet geometry: a 2-D planar inlet was used in [5] and an inward turning inlet was used in [12], the latter had a highly swept leading edge, which is beneficial to supersonic spillage, so it had a larger maximum self-starting internal contraction ratio. Furthermore, a different way for imposing unstart mechanically is also a probable factor to bring the difference. Studies by Goldberg and Hefner [9] and Smart [11] have a close self-starting internal contraction ratio, though the former has a higher Mach number at cowl-lip section, because boundary-layer factor was considered in [9], and boundary layer covered 78% height of cowl-lip section at this operation.

According to the data of [10–12], it is found that there seems to be a polynomial law between the maximum self-starting internal contraction ratio and the average Mach number of the cowl-lip section, given as the following equation:

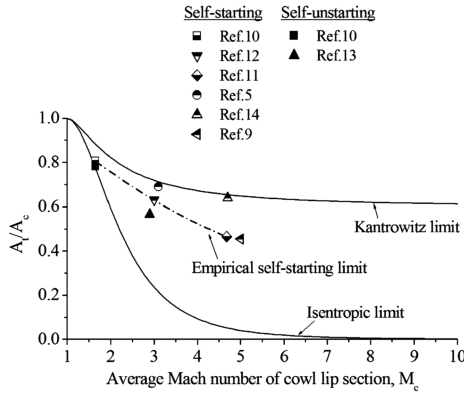


Fig. 1 Self-starting limits for supersonic inlets.

$$\left(\frac{A_c}{A_t}\right)_{\text{empirical}} = 0.933 + \frac{M_c}{6.87} + \frac{M_c^2}{40.9} \quad (1.65 \leq M_c \leq 4.68) \quad (2)$$

The preceding fit is plotted by the dash-dot line in Fig. 1.

### III. Conclusions

Equation (2) presents the approximate relationship of maximum self-starting internal contraction ratio and average Mach number of cowl-lip section, which is probably useful for designing a supersonic inlet. Self-starting characteristics of an inlet can be influenced by many factors as inlet geometry, boundary layer, Reynolds number, etc., but it is certain that additional ways helping self-starting (like bleed, bypass ducts, fluid injection, and so on) should be considered if an inlet has a larger internal contraction ratio than the value limited by Eq. (2), especially when the Mach number of the cowl-lip section is lower.

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